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MATHEMATICS
TARGET IIT JEE

Solution Slot -3
(English Medium)

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STRAIGHT LINE

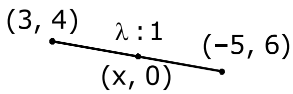
EXERCISE – I
HINTS & SOLUTIONS

Sol.1 D

since the points are collinear option D is correct

Sol.2 A

$$\frac{-5\lambda+3}{\lambda+3} = x,\ \frac{6\lambda-4}{\lambda+1} = 0$$

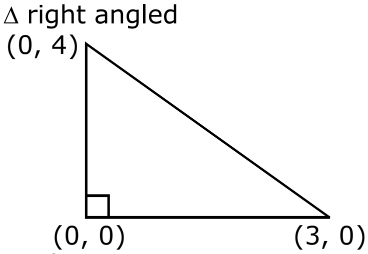


$$\Rightarrow \lambda = \frac{2}{3}$$

$$x = \frac{-5,\frac{2}{3}+3}{\frac{2}{3}+1} = \frac{-10+9}{2+3} = -\frac{1}{5}$$

Sol.3 C

Δ right angled

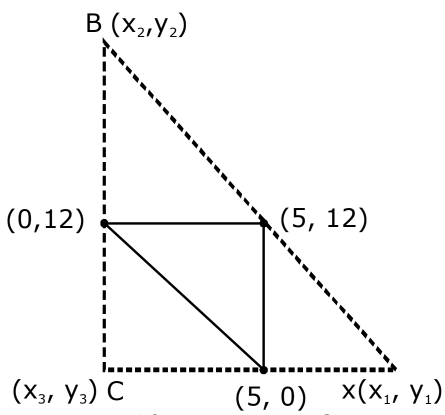


$\Rightarrow$ circum centre

= mid point of hypotaneous = $\left(\frac{3}{2},2\right)$

Sol.4 A

$$\begin{cases} x_1 + x_3 = 10 \ , & y_1 + y_3 = 0 \\ x_2 + x_3 = 0 \ , & y_2 + y_3 = 24 \\ x_1 + x_2 = 10 \ , & y_2 + y_2 = -24 \end{cases}$$



$x_1 = x_2 = 10,\ y_1 - y_2 = -24$
$x_1 = 10,\ y_1 = 0$
$x_2 = 0,\ y_2 = 24$
$x_3 = 0,\ y_3 = 0$

$$\left.\begin{matrix} x_1 = 10 \ , & y_1 = 0 \\ x_2 = 0 \ , & y_2 = 24 \\ x_3 = 0 \ , & y_3 = 0 \end{matrix}\right\} \Rightarrow \begin{matrix} A(10,0) & \text{on} & x-\text{axis} \\ B(a24) & \text{on} & y-\text{axis} \\ C(40) & \text{is} & \text{origin} \end{matrix}$$

ΔABC is right angled $\Rightarrow$ orthocentre is (0, 0)

Sol.5 D

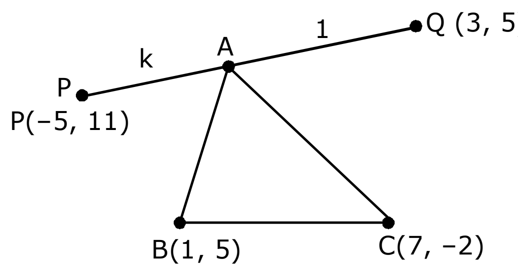
$$\Delta = \left|\frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{vmatrix}\right|$$

$$\xrightarrow{R_1\to R_1+R_3} \left|\begin{vmatrix} 0 & 0 & 2 \\ -a\sin\theta & b\cos\theta & 1 \\ -a\cos\theta & -b\sin\theta & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}.\ 2\ (ab\sin^2\theta + ab\cos^2\theta) = ab$$

Sol.6 C

$$\left(\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right)$$



$$\frac{1}{2}\begin{vmatrix} \frac{3k-5}{k+1} & \frac{5k+1}{k+1} & 1 \\ 1 & 5 & 1 \\ 7 & -2 & 1 \end{vmatrix} = |2|$$

$$\Rightarrow 1.\ (-2-3) - 1.\left(\frac{-6k+10}{k+1} - \frac{35k+7}{k+1}\right)$$

$$+\left(\frac{15k-25}{k+1} - \frac{5k+1}{k+1}\right) = \pm 4$$

$\Rightarrow$ 6k – 10 + 35 k + 7 + 15k – 25 – 5k – 1 = ± 4 + 37 (k + 1)

$\Rightarrow$ 51 k – 29 = 41 k + 41 or 51 k – 29 = 33k + 33

$\Rightarrow$ 10 k = 70 or 18 k = 62

$k = 7$ $\qquad$ $k = \frac{31}{9}$

Sol.7 C

Let centroid is (h, k)

then $h = \dfrac{\cos\alpha + \sin\alpha + 1}{3}$ & $k = \dfrac{\sin\alpha - \cos\alpha + 2}{3}$

$\cos\alpha + \sin\alpha = 3h - 1$ & $\sin\alpha - \cos\alpha = 3k - 2$

squaring & adding

$2 = (3h - 1)^2 + (3k - 2)^2$ Locus of (h, k)

$\Rightarrow (3x - 1)^2 + (3k - 2)^2 = 2$

$\Rightarrow 3(x^2 + y^2) - 2x - 4y + 1 = 0$

Sol.8 D

(2a, 3a), (3b, 2b) & (c, c) are collinear

$$\Rightarrow \begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0$$

$\Rightarrow$ (3bc – 2bc) – (2ca – 3ca) + (4ab – 9ab) = 0

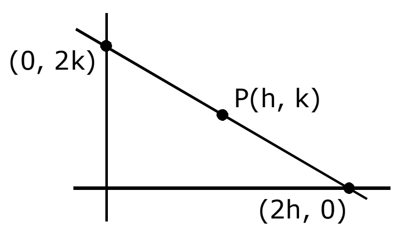
$\Rightarrow$ bc + ca + 5ab = 0

$$\Rightarrow \frac{2}{2} \cdot \frac{5}{c} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{2}{\left(\frac{2C}{5}\right)} = \frac{1}{a} + \frac{1}{b}$$

$\Rightarrow$ a, $\dfrac{2C}{5}$, b are in H.P.

Sol.9 B

P is a mid point AB



AB = 10 units

$(2h)^2 + (2k)^2 = 10^2$

$h^2 + k^2 = 25$

Locus of (h, k)

$x^2 + y^2 = 2\ 5$

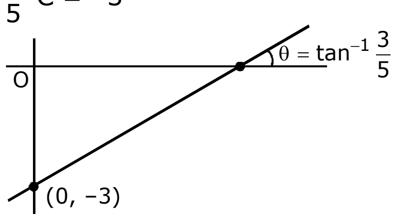
Sol.10 A

$\theta = \tan^{-1}\dfrac{3}{5}$ C = –3

$\tan\theta = \dfrac{3}{5}$

$y = \dfrac{3}{5}x - 3$

3x – 5y–15=0



Sol.11 D

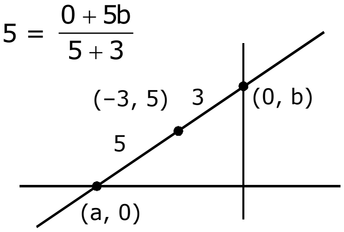
$-3 = \dfrac{3a+0}{5+3}$, $5 = \dfrac{0+5b}{5+3}$

$\Rightarrow$ a = –3, b = 8

$\dfrac{x}{-8} + \dfrac{y}{8} = 1$

– x + y = 8

x – y + 8 = 0

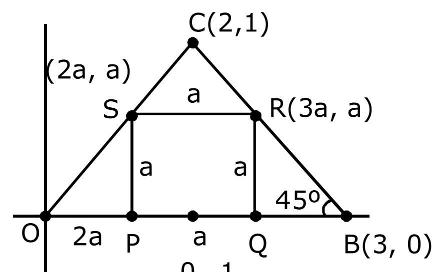


Sol.12 D

Let side of square is a units

equation of OC is 2y = x

S(2a, a) $\Rightarrow$ R(3a, a)



Slope $m_{BC} = \dfrac{0-1}{3-2} = -1$

$\Rightarrow \angle B = 45^\circ$ in ΔQBR

QB = a

OB = OP + PQ = QB

$3 = 2a + a + a \Rightarrow a = \dfrac{3}{4}$

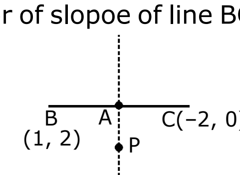
$P\left(\dfrac{3}{2}, 0\right)$, $Q\left(\dfrac{9}{4}, 0\right)$, $R\left(\dfrac{9}{4}, \dfrac{3}{4}\right)$ & $S\left(\dfrac{3}{2}, \dfrac{3}{4}\right)$

Sol.13 C

Perpendicular bisector of slopoe of line BC

$m_{BC} = \dfrac{2-0}{1+2} = \dfrac{2}{3}$

$m_{AP} = \dfrac{-3}{2}$



$A = \left(\dfrac{1-2}{2}, \dfrac{2+0}{2}\right) \Rightarrow \left(-\dfrac{1}{2}, 1\right)$

$y - 1 = \dfrac{-3}{2}\left(x + \dfrac{1}{2}\right) \Rightarrow 4y - 4 = -6x - 3$

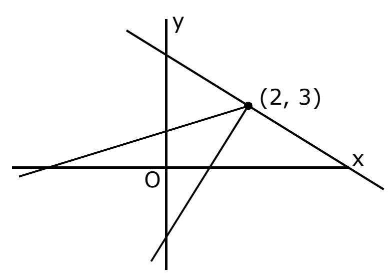
$\Rightarrow$ 6x + 4y = 1

locus of P

Sol.14 C

Equation y – 3 = m (x – 2)

cut the axis at



$\Rightarrow$ y = 0 & x = $\dfrac{2m-3}{m}$

$\Rightarrow$ x = 0 & y = – (2m – 3)

Area Δ = 12 = $\left|\dfrac{1}{2}.\dfrac{(2m-3)}{m}\{-(2m-3)\}\right|$

$(2m – 3)^2 = \pm 24m$

$4m^2 – 12m + 9 = 24m$

or $4m^2 – 12m + 9 = – 24m$

$4m^2 – 3y\ m + 9 = 0$

D > 0

or $4m^2 + 12m + 9 = 0$

$(2m + 3)^2 = 0$

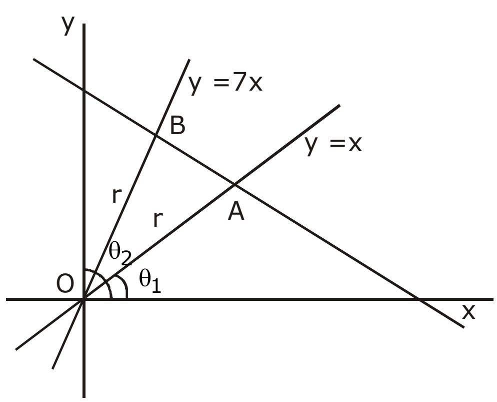
two distinct root of m

no. of values of m is 3.

Sol.15 D

OA line y = x, $m_1 = \tan\theta_1 = 1$

OB line y = 7m, $m_2 = \tan\theta_2 = 7$



A, B lies in I^{st} quadrant

OA = OB = r (let)

OA line $\dfrac{x}{\cos\theta_1} = \dfrac{y}{\sin\theta_1} = r \Rightarrow \dfrac{x}{\frac{1}{\sqrt{2}}} = \dfrac{y}{\frac{1}{\sqrt{2}}} = r$

A $\left(\dfrac{r}{\sqrt{2}}, \dfrac{r}{\sqrt{2}}\right)$

OB line $\dfrac{x}{\frac{1}{5\sqrt{2}}} = \dfrac{y}{\frac{7}{5\sqrt{2}}} = r \Rightarrow B.\left(\dfrac{r}{4\sqrt{2}}, \dfrac{7r}{5\sqrt{2}}\right)$

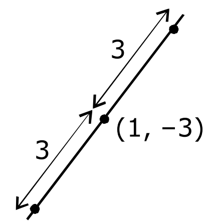
Slope $m_{AB} = \dfrac{\frac{7r}{5\sqrt{2}} - \frac{r}{\sqrt{2}}}{\frac{1}{5\sqrt{2}} - \frac{r}{\sqrt{2}}} = \dfrac{7r-5r}{r-5r} = \dfrac{2}{-4} = -\dfrac{1}{2}$

Sol.16 B

2x + 3y + 7 = 0

$\tan\theta = \dfrac{-2}{3} \Rightarrow \sin\theta = \dfrac{2}{\sqrt{13}}, \cos\theta = \dfrac{-3}{\sqrt{13}}$

$\dfrac{x-1}{\frac{-3}{\sqrt{13}}} = \dfrac{y+3}{\frac{2}{\sqrt{13}}} = \pm 3$



$\left(1 - \dfrac{9}{\sqrt{13}}, -3 + \dfrac{6}{\sqrt{13}}\right)$

or $\left(1 + \dfrac{9}{\sqrt{13}}, -\dfrac{3-6}{\sqrt{13}}\right)$

Sol.17 A

y – x + 5 = 0 , $\sqrt{3}$ x – y + 7 = 0

$m_1 = 1$ $\quad m_2 = \sqrt{3}$

$\theta_1 = 45^\circ$ $\quad \theta_2 = 60^\circ$

$\theta = 60^\circ – 45^\circ = 15^\circ$

Aliter $\tan\theta = \dfrac{\sqrt{3}-1}{1+\sqrt{3}} = \dfrac{4-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$

$\Rightarrow \theta = 15^\circ$

Sol.18 D

$\perp$ to 3x + y = 3, passes (2, 2)

m = + $\dfrac{1}{3}$ & (2, 2)

y – 2 = + $\dfrac{1}{3}$ (x – 2)

$\Rightarrow -x + 3y = 4 \Rightarrow \dfrac{x}{-4} + \dfrac{y}{\frac{4}{3}} = 1 \Rightarrow b = \dfrac{4}{3}$

Sol.19 C

required line should be

ax + by + λ = 0 satsify (c, d)

ac + bd + λ = 0 $\Rightarrow \lambda$ = – (ac + bc)

ax + by – (ac + bc) = 0

$\Rightarrow$ a (x – c) + b (y – d) = 0

Sol.20 A

L_1 : 2x + 3y – 4 = 0

L_2 : 6x + 96 + 8 = 0 , P(8, – 9)

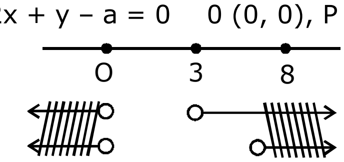
L_1(P) = 2.8 – 3.9 – 4 = 16 – 27 – 4 = – 15 < 0

L_2(O) = 48 – 81 + 8 + 8 = – 25 < 0

point (8, –9) lies same side of both lines.

Sol.21 C

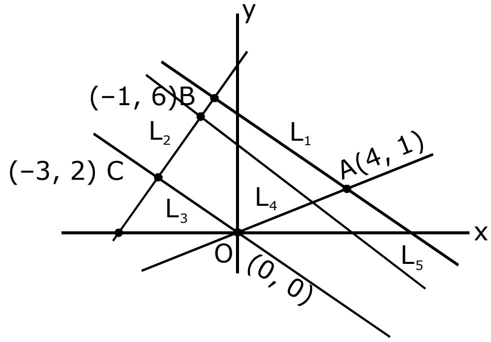
L_1 ; 2x + y – a = 0 0 (0, 0), P (3, 2)



$L_2 : x - 3y + a = 0$
$\Rightarrow\ L_1(0)\,L_1(P) > 0\ \ \&\ L_2(0)\,L_2(P) > 0$
$-a(8 - a) > 0\ \ \&\ \ a(-3 + a) > 0$
$a(a - 8) > 0\ \ \&\ a(a - 3) > 0$
$a \in (-\infty, 0) \cup (8, \infty)\ \&\ a \in (-\infty, 0) \cup (3, \infty)$
$\Rightarrow a \in (-\infty, 0) \cup (8, \infty)$

Sol.22 A

$L_1 : x + y = 5,\ L_2 : y - 2x = 8$
$L_3 : 3y + 2x = 0,\ L_4 : 4y - x = 0$
$L_5 : (3x + 2y) = 6$

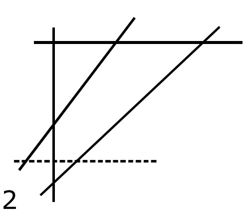


vertices of quadrilateral
$0(0, 0),\ A(4, 1),\ B(-1, 6),\ C(-3, 2)$
$L_5(0) = -6 < 0$
$L_5(A) = 12 + 2 - 6 = 8 > 0$
$L_5(B) = -3 + 12 - 6 = 3 > 0$
$L_5(C) = -9 + 4 - 6 = -11 < 0$
O & C points are same side
& A & B points are other same side w.r.t to L_5
So L_5 divides the quadrilateral in two quadrialteral

Aliter :
If abscissa of A is less then abscissa of B
$\Rightarrow$ A lies left of B
otherwise A lies right of B

Sol.23 B

P(a, 2) lies between
$L_1 : x - y - 1 = 0$ &
$L_2 : 2(x - y) - 5 = 0$
Method-I
$L_1(P)\,L_2(P) < 0$
$(a - 3)(2a - 9) < 0$
$\Rightarrow$ P(a, 2) lies on y = 2
intersection with given lines



$x = 3\ \ \&\ \ x = \dfrac{9}{2}$

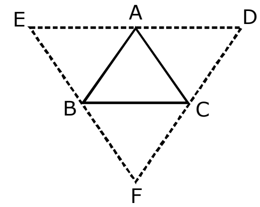
$a > 3\ \ \&\ \ a < \dfrac{9}{2}$

(gemetrically)

$a \in \left(3, \dfrac{9}{2}\right)$

Sol.24 C

$A(x_1, y_1),\ B(x_2, m\ y_2),\ C(x_3, y_3)$



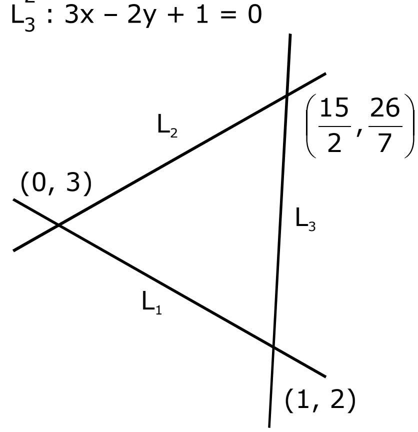
only three sides can be made parallel to corresponding sides of triangle passing through vertex of triangle respectively
$\Rightarrow$ So no. of IIgrams is 3.

Sol.25 D

P(1, 0), Q(–1, 0), R(2, 0), Locus of s (h, k) if $SQ^2 + SR^2 = 2SP^2$
$\Rightarrow (h + 1)^2 + k^2 + (h - 2)^2 + k^2$
$= 2(h - 1)^2 + 2k^2$
$\Rightarrow h^2 + 2h + 1 + h^2 - 4h - 4 = 2h^2 - 4h + 2$
$\Rightarrow 2h + 3 = 0$ Locus of s(h, k)
$\Rightarrow 2x + 3 = 0$
Parallel to y-axis.

Sol.26 B

$L_1 : x + y - 3 = 0,$
$L_2 : x - 3y + 9 = 0$
$L_3 : 3x - 2y + 1 = 0$



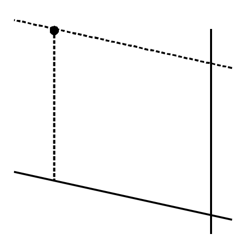
$$\Delta = \frac{1}{2}\begin{vmatrix} \frac{15}{7} & \frac{26}{7} & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\frac{15}{7}(3-2) + 0 + 1\left(\frac{26}{7} - 3\right)\right]$$

$$= \frac{1}{2}\left[\frac{15}{7} + \frac{5}{7}\right] = \frac{10}{7} \text{ sq.units}$$

Aliter : by parallelogram

$$\Delta = \frac{1}{2}\left|\frac{(c_1 - c_2)(d_1 - d_2)}{(m_1 - m_2)}\right|$$

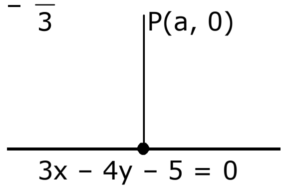


Sol.27 D

$m = \frac{3}{4} \Rightarrow m_{PQ} = -\frac{4}{3}$

equation of PQ

$y - 5 = -\frac{4}{3}x$



$4x + 3y - 15 = 0$

$\Rightarrow 25x = 75$

& $3x - 4y - 5 = 0 \Rightarrow x = 3$ & $y = 1$

Q(3, 1)

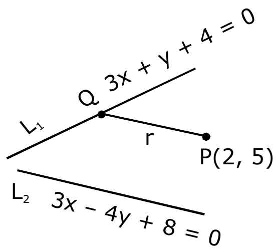
Sol.28 C

$m_{PQ} = \frac{3}{4} = \tan\theta$

$\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$

Let PQ = r

equation PQ



$\frac{x-2}{4/5} = \frac{y-5}{3/5} = \frac{4}{5}$

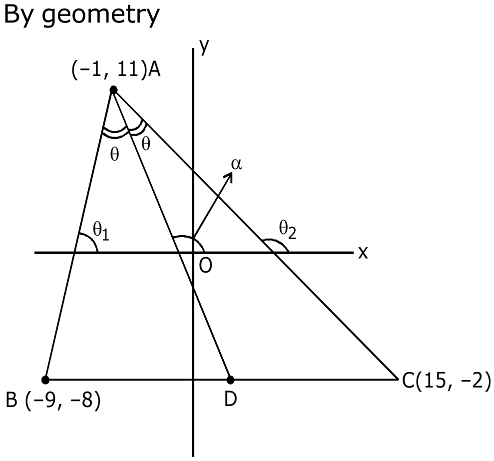
$\Rightarrow x = \left(\frac{4r}{3}+2\right)$ & $y = \left(\frac{3r}{5}+5\right)$ lies on L_1

$3\left(\frac{4r}{5}+2\right)+\left(\frac{3r}{5}+5\right)+4=0 \quad \Rightarrow \frac{154}{5} = -15$

$\Rightarrow r = -5 \Rightarrow |r| = 5$ units

Sol.29 B

By geometry



Angle bisector of A is origin containing

line AB : $19x - 8y + 107 = 0$

Line AC : $-13x - 16y + 163 = 0$

$$\frac{19x - 8y + 107}{\sqrt{19^2 + 8^2}} = \frac{-13x - 16y + 163}{\sqrt{13^2 + 16^2}}$$

$\{19^2 + 8^2 = 13^2 + 16^2 = 425$

$\Rightarrow 32x + 8y - 56 = 0 \Rightarrow 4x + y = 7$

Aliter :

$m_{AB} = \frac{19}{8} = \tan\theta_1$, $m_{AC} = \tan\theta_2 = \frac{-13}{16}$

$$\tan 2\theta = \left|\frac{\frac{19}{8}+\frac{13}{16}}{1-\frac{19}{8}\cdot\frac{13}{6}}\right| = \left|\frac{-136}{13}\right|$$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{136}{13}$ {θ is acute $\tan\theta > 0$

$\Rightarrow 68\tan^2\theta + 13\tan\theta - 68 = 0 \Rightarrow \tan\theta = 0.9$

$\alpha = \theta + \theta_1$

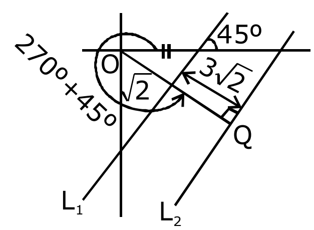
$\tan\alpha = \frac{\tan\theta + \tan\theta_1}{1 - \tan\theta\tan\theta_1}$

equation is $(y - 11) = \tan\alpha\,(x + 1)$

Sol.30 D

$OP = \sqrt{2}$, $PQ = 3\sqrt{2}$

$OQ = 4\sqrt{2}$



OQ makes angle with (+) x-axis in anti clockwise $\theta = 270° + 45°$

equation L_2

$x\cos\theta + y\sin\theta = 4\sqrt{2}$

$x\cos(270° + 45°) + y\sin(270°+45°) = 4\sqrt{2}$

$x\sin 45° + y(-\cos 45°) = 4\sqrt{2}$

$x - y = 8$

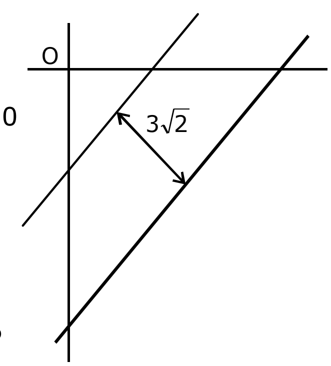
Aliter :

$y - x + 2 = 0$

$\Rightarrow x - y - 2 = 0$

Parallel lines

$x - y + \lambda = 0$



$3\sqrt{2} = \left|\frac{\lambda+2}{\sqrt{2}}\right|$

$\Rightarrow \lambda + 2 = \pm 6$

$\Rightarrow \lambda = -8, 4$

Line shift to (+) x-axis

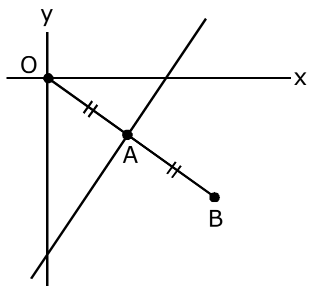
So line is $x - y - 8 = 0$

Sol.31 B

Point of reflection of (0, 0)
w.r.t. to $4x - 2y - 5 = 0$

$$OA = \left|\frac{-5}{\sqrt{4^2+2^2}}\right| = \frac{2}{2\sqrt{5}}$$

$$= \frac{\sqrt{5}}{2} = AB$$



equtaion of line OB

$$\frac{x-0}{-\frac{2}{\sqrt{5}}} = \frac{y-0}{\frac{1}{\sqrt{5}}} = \pm\sqrt{5}$$

$\Rightarrow OB = \sqrt{5}$

$x = \mp\sqrt{2}$, $y = \pm 1$ $\quad \Rightarrow$ B (2, –1)

Aliter :

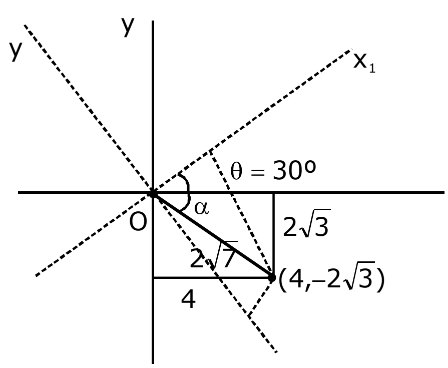
Image of origin w.r. to line

$$\frac{x-0}{4} = \frac{y-0}{-2} = \frac{-2(4.0-2.0-5)}{4^2+(-2)^2}$$

$$\Rightarrow \frac{x}{4} = \frac{y}{-2} = \frac{10}{20} \Rightarrow x = 2,\ y = -1,\ B(2, -1)$$

Sol.32 B

First position



$(4, -2\sqrt{3}) = (4\cos(-\alpha),\ r\sin(-\alpha))$

$r\cos\alpha = 4$

$r\sin\alpha = +2\sqrt{3}$

& $\sin\theta^{o} = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

Last position w.r.t is x′

$(r\cos(-\theta - a,\ r\sin(-\theta - \alpha))$

$= (r\cos(\theta + \alpha),\ -r(\sin(\theta + \alpha))$

$= ((4\cos\theta\cos\alpha - r\sin\alpha\sin\alpha)),$
$\quad m(-r\cos\alpha\sin\theta - r\sin\alpha\cos\theta)$

$$= \left(\left(4.\frac{\sqrt{3}}{2} - 2\sqrt{3}, \frac{1}{2}\right), \left(-4.\frac{1}{2} - 2\sqrt{3}.\frac{\sqrt{3}}{2}\right)\right)$$

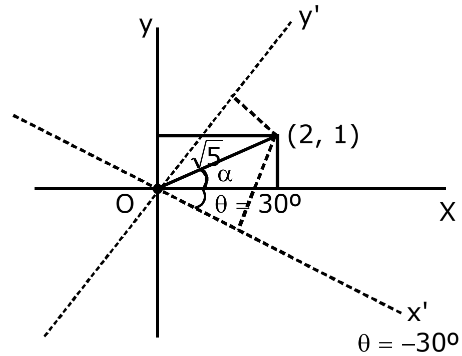
$= ((2\sqrt{3} - \sqrt{3}),\ (-2 - 3)) = (\sqrt{3}, -5)$

Sol.33 B

Before rotation

$(2, 1) = (4\cos\alpha,\ r\sin\alpha)$

$r\cos\alpha = 2$, $r\sin\alpha = 1$



new position

$\Rightarrow x' = 4\cos\alpha\cos\alpha - r\sin\alpha\sin\theta$

$$= 2.\frac{\sqrt{3}}{2} + 2.\left(\frac{-1}{2}\right) = \frac{\sqrt{3}-2}{2}$$

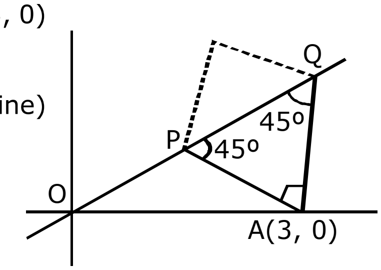
$$(x', y') = \left(\frac{2\sqrt{3}+1}{2}, \frac{\sqrt{3}-2}{2}\right)$$

Sol.34 D

$x = 2y$, A(3, 0)

$y = m(x - 3)$

$m_1 = \frac{1}{2}$ (given line)



$$\tan 45^{o} = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right|$$

$$\Rightarrow \left|1 + \frac{m}{2}\right| = \left|m - \frac{1}{2}\right|$$

$$\Rightarrow \left(1 + \frac{m}{2}\right) = \left(m - \frac{1}{2}\right) \text{ or } \frac{3m}{2} = -\frac{1}{2}$$

$\Rightarrow m = 3 \qquad m = -\frac{1}{3}$

lines are $y = 3(x - 3)$

$\Rightarrow 3x - y - 9 = 0$ & $y = -\frac{-1}{3}(x - 3)$

$\Rightarrow x + 3y - 3 = 0$

Sol.35 D

$(p + 2q)x + (p - 3q)y = p - q$

$px + py - p + 2qx - 3qy + q = 0$

$p(x + y - 1) + q(2x - 3y + 1) = 0$

passing through intersection of

$x + y - 1 = 0$ & $2x - 3y + 1 = 0$ is $\left(\frac{2}{5}, \frac{3}{5}\right)$

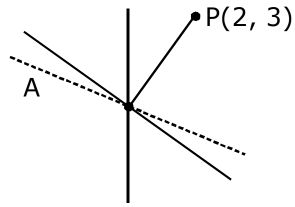
Sol.36 A

PM is maximum if required line $\perp$ intersection of

$3x + 4y + 6 = 0$

$\Rightarrow (-2, 0)$

$x + y + 2 = 0$



$m_{AP} = \frac{3-O}{2+2} = \frac{3}{4}$

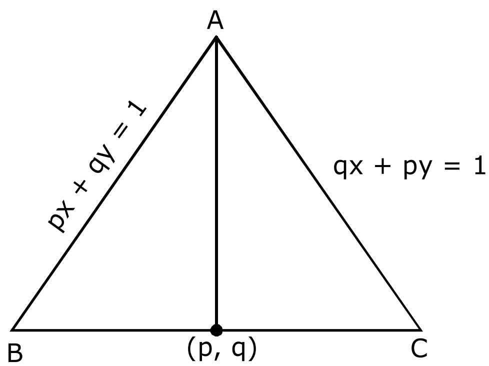
Slope $m = -\frac{4}{3}$

$y - 0 = -\frac{4}{3}(x + 2) \Rightarrow 4x + 3y + 8 = 0$

Sol.37 C

$L_1 : Px + qy = 1$

$L_2 : qx + py = 1$



$L_1 + \lambda L_2 = 0$

$(px + qy - 1) + \lambda (qx + py - 1) = 0$

$\Rightarrow \lambda = \frac{(p^2+q^2-1)}{(2pq-1)} \Rightarrow (2pq-1)(px + qy - 1)$

$= (p^2 + q^2 - 1)(qx + py - 1)$

Sol.38 C

$2x^2 + 4xy / py^2 + 4x + 4x + qy + 1 = 0$

$a = 2, b = -p, c = 1, f = -\frac{q}{2}, y = 2, h = 2$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$\Rightarrow -2p + 4q - \frac{q^2}{2} + 4P - 4 = 0$

$\Rightarrow 2P + 4q - \frac{q^2}{2} - 4 = 0$

$\perp \Rightarrow a + b = 0$

$\Rightarrow 4 + 4q - \frac{q^2}{2} - 4 = 0 \qquad 2 - p = 0$

$\Rightarrow q\left(4 - \frac{q}{2}\right) = 0 \qquad p = 2$

$\Rightarrow q = 0, q = 8$

Sol.39 A

$5x^2 - 7xy - 3y^2 = 0$

we know if given lines $ax^2 + 2hxy + by^2$ then

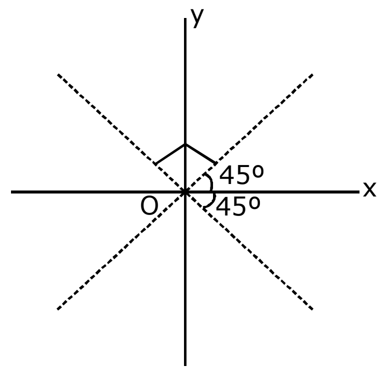
$\perp$ lines are $bx^2 - hxy + ay^2 = 0$

$\Rightarrow -3x^2 - (-7xy) + 5y^2 = 0$

$\Rightarrow 3x^2 - 7xy - 5y^2 = 0$

Sol.40 D

$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ & $(x + ky) = 1$



$\Rightarrow 5x^2 + 12xy - 6y + 4x(1) + 3(1)^2 = 0$

$\Rightarrow 5x^2 + 12xy - 6y^2 + 4x(x + ky) + 2y(x + ky) + 3(x + ky)^2 = 0$

These lines are equally inclined to coordinate axis.

$\Rightarrow m_1 + m_2 = 0 \Rightarrow -\frac{2h}{b} = 0 \Rightarrow h = 0$

$5(k + 1) = 0 \qquad \Rightarrow k = -1$

& both line perpendicular also $a + b = 0$

$12 + 3k^2 - 2k - 6 = 0$

$3k^2 - 2k + 6 = 0$

$D < 0 \qquad \Rightarrow$ K is not real

real k doesn't exist

Sol.41 B

$$\begin{vmatrix} \sin^2 A & \sin A & 1 \\ \sin^2 B & \sin B & 1 \\ \sin^2 C & \sin C & 1 \end{vmatrix} = 0$$

$\Rightarrow (\sin A - \sin B)(\sin B - \sin C)(\sin C - \sin C) = 0$

$\Rightarrow A = B$ or $B = C$ or $C = A$

any two angles are equal $\Rightarrow \Delta$ is isosceles

Sol.42 B

P lies on $2x - y + 5 = 0$

$|PA - PB|$ is maximum

we know

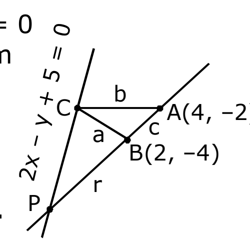
$b < a + c$

$b - a < c$

If $b - a = c$

then $(P - PB)$ is max.

$\Rightarrow$ PBA colinear



Slope $m_{AB} = 1 = \tan\theta$ If PB = r

$$\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y+4}{\frac{1}{\sqrt{2}}} = r \Rightarrow x = \frac{r}{\sqrt{2}} + 2,\ y = \frac{r}{\sqrt{2}} - 4$$

Satisfy given equation

$$2\left(\frac{r}{\sqrt{2}} + 2\right) - \left(\frac{r}{\sqrt{2}} - 4\right) + 5 = 0$$

$$2\frac{r}{\sqrt{2}} + 4 - \frac{r}{\sqrt{2}} + 4 + 5 = 0$$

$$\frac{r}{\sqrt{2}} = -13 \quad \Rightarrow r = -13\sqrt{2}$$

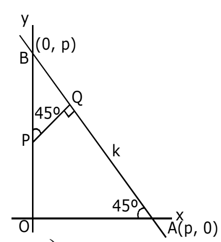
$$P\left(\frac{-13\sqrt{2}}{\sqrt{2}} + 2, \frac{-13\sqrt{2}}{\sqrt{2}} - 4\right) \equiv (-11, -17)$$

Sol.43 D

x + y = p

Let Q divides AB in k : 1

$$\frac{\Delta Q}{QB} = \frac{k}{1}$$



$$Q\left(\frac{p}{k+1}, \frac{pk}{k+1}\right),\ m_{PQ} = 1$$

line PQ . $y - \frac{kp}{k+1} = \left(x - \frac{p}{k+1}\right)$ (If cut y-axis)

then (x=0 put) $\Rightarrow y = \frac{(k-1)p}{(k+1)}$, $p\left(0, \frac{pk-p}{k+1}\right)$

$$PQ = BQ = \sqrt{\left(\frac{p}{k+1}\right)^2 + \left(\frac{pk}{k+1} - \frac{pk}{k+1} + \frac{p}{k+1}\right)^2} = \frac{\sqrt{2}pk}{k+1}$$

Area $\Delta APQ = \frac{3}{8}\ \Delta OAB = \frac{3}{8} \cdot \frac{1}{2}p^2 = \frac{3}{16}p^2$

$$\Rightarrow \frac{1}{2}\ \frac{\sqrt{2}pk}{(k+1)} \cdot \frac{\sqrt{2}p}{(k+1)} = \frac{3}{16}\ p^2$$

$\Rightarrow 16k = 3(k+1)^2 \Rightarrow 3k^2 + 6k + 3 = 16k$

$\Rightarrow k = 3$ $\quad k = \frac{1}{3}$ is reject

($\because$ P lies on OB only)

Sol.44 B

$L_1 : x + \sqrt{3}\,y = 2$, $L_2 : ax + by = 1$, q = 45º,

$L_3 = y\ \sqrt{3}\,x$

$$\begin{vmatrix} 1 & \sqrt{3} & -2 \\ a & b & -1 \\ \sqrt{3} & -1 & 0 \end{vmatrix} = 0$$

$\Rightarrow \sqrt{3}\ (-\sqrt{3} + 2b) + (-1 + 2a) = 0$

$\Rightarrow a + \sqrt{3}\,b = 2$ ....(i)

$m_1 = \frac{-1}{\sqrt{3}}$, $m_2 = -\frac{a}{b}$

$$\tan 45º = \left|\frac{-\frac{1}{\sqrt{3}} + \frac{a}{b}}{1 + \frac{a}{\sqrt{3}b}}\right|$$

$\Rightarrow |a + \sqrt{3}\,b| = |\sqrt{3}\,a - b|$

$\Rightarrow (a + \sqrt{3}\,b)^2 + 2\sqrt{3}\,ab = 3a^2 + b^2 - 2\sqrt{3}\,ab$

$\Rightarrow a^2 + b^2 - 2\sqrt{3}\,ab$ ....(ii)

squaring (i) & adding (ii)

$2a^2 + ab^2 = 4 \Rightarrow a^2 + b^2 = 2$

Sol.45 D

$L_1 : 2x - 3y - 6 = 0$

$L_2 : 3x - y + 3 = 0$

$L_3 : 3x + 4y - 12 = 0$ $\quad P(a, 0), Q(0, \beta)$

By geometry origin lies in Δ

$L_1(0) < 0$ & $L_2(0) > 0$ $L_3(0) < 3$

$\Rightarrow L_1(P) \le 0$ & $L_2(P) \ge 0$ & $L_3(P) \le 0$

$\alpha - 3$ & $a + 1 \ge 0$ & $a \le 4$

$\Rightarrow a \in [-1, 3]$

$\Rightarrow L_1(Q) \le 0$ & $L_2(Q) \ge 0$ & $L_3(Q) \le 0$

$-3\beta - 6 \le 0$ & $-b + 3 \ge 0$ & $4\beta - 12 \le 0$

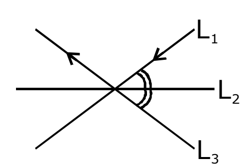
$\beta \ge -2$ & $\beta \le 3$ $\beta \le 3$ & $\beta \le 3 \Rightarrow \beta \in [-2, 3]$

Sol.46 C

$L_1 : x - 7y + 5 = 0 \quad \Rightarrow m_1 = \frac{1}{7}$

$L_2 : x + 3y - 2 = 0 \quad \Rightarrow m_2 = -\frac{1}{3}$

$L_3 : (x + 3y - 2) + \lambda(x - 7y + 5) = 0$



$$\tan\theta = \left|\frac{\frac{1}{7}+\frac{1}{3}}{1-\frac{1}{21}}\right| = \frac{1}{2} \quad \tan\theta = \frac{m_3+\frac{1}{3}}{1-\frac{m_3}{3}} = \frac{1}{2}$$

$$m_3 = -\frac{(1+\lambda)}{(3-7\lambda)} = -1 \quad \Rightarrow m_3 = -1$$

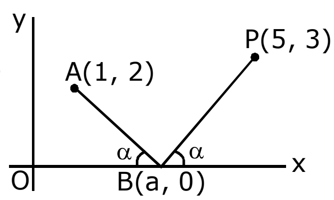
$$1 + \lambda = 3 - 7\lambda \qquad \Rightarrow \lambda = \frac{1}{4}$$

$$\Rightarrow x + 3y - 2 + \frac{1}{4}(x - 7y + 5) = 0$$

$$\Rightarrow 5x + 5y - 3 = 0$$

Sol.47 A

$m_{AB} + m_{PB} = 0$



$$\frac{2}{1-a}+\frac{3}{5-a}=0$$

$$\Rightarrow a = \frac{13}{5}$$

$$m_{AB} = \frac{2}{1-\frac{13}{5}} = \frac{10}{-8} = \frac{5}{-4}$$

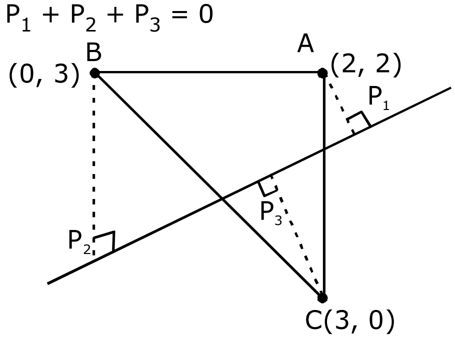
equation of AB

$$\Rightarrow y - 2 = -\frac{-5}{4}(x-1) \quad 5x + 4y = 13$$

Sol.48 D

Let a line $ax + by + c = 0$

$P_1 + P_2 + P_3 = 0$



$$\frac{3a+c}{\sqrt{a^2+b^2}} + \frac{3b+c}{\sqrt{a^2+b^2}} + \frac{2a+2b+c}{\sqrt{a^2+b^2}} = 0$$

$$5a + 5b + 3c = 0$$

$$a\left(\frac{5}{3}\right) + b\left(\frac{5}{3}\right) + C = 0$$

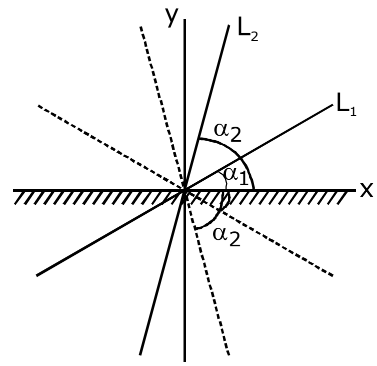
$$\Rightarrow \left(\frac{5}{3},\frac{5}{3}\right) \text{ satisfy the given line}$$

$$\Rightarrow \text{fix point is } \left(\frac{5}{3},\frac{5}{3}\right) \text{ which is centroid of } \Delta ABC$$

Sol.49 A

$$ax^2 + 2hxy + by^2 = 0$$

$$m_1 + m_2 = \frac{-2h}{b},\ m_1m_2 = \frac{a}{b}$$



Relation of slopes of image lines

$$(m_1' + m_2') = -(m_1 + m_2)$$

$$= -\left(\frac{-2h}{b}\right) = \frac{2h}{b} \qquad \{m_1' = \tan(\alpha_1)$$

$$m_1'm_2' = (-m_1)(-m_2)$$

$$= m_1m_2 = \frac{a}{b}$$

$$\left(\frac{y}{x}\right)^2 - (m_1' + m_2')\left(\frac{y}{x}\right) + m_1'm_2' = 0$$

$$\Rightarrow \left(\frac{y}{x}\right)^2 - \frac{2h}{b}\left(\frac{y}{x}\right) + \frac{a}{b} = 0$$

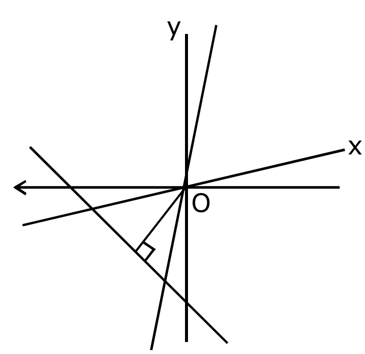
$$\Rightarrow by^2 - 2hxy + ax^2 = 0$$

$$\Rightarrow ax^2 - 2hxy + by^2 = 0$$

Sol.50 D

$$x^2 - 4xy + y^2 = 0,\ x + y + 4\sqrt{6} = 0$$

angle bisector of given pair of st. lines



$\frac{x^2 - y^2}{a - b} = \frac{xy}{h} \Rightarrow \frac{x^2 - y^2}{1 - 1} = \frac{xy}{-2}$

$\Rightarrow$ $x^2 - y^2 = 0$

$\Rightarrow$ $(x + y)(x - y) = 0$

$x + y = 0$ is || to third side

altitude $\equiv$ angle bisector $\Rightarrow$ isosceles Δ

Now $\tan\theta = \left|\frac{2\sqrt{h^2 ab}}{a + b}\right| = \left|\frac{2\sqrt{4 - 1}}{2}\right| = \sqrt{3}$

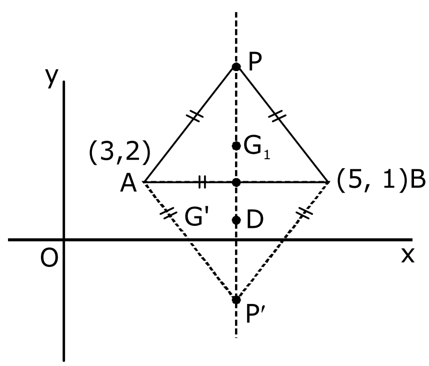
$\Rightarrow \theta = 60^o$

$\Rightarrow$ angle between two equal sides is 60^o

$\Rightarrow$ equiliteral Δ

Sol.51 D

$D\left(4, \frac{3}{2}\right)$, AB = $\sqrt{4 + 1} = \sqrt{5}$



$PD = \sqrt{5 - \frac{5}{4}} = \sqrt{\frac{15}{2}}$

G.D. = $\frac{1}{3} . \frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{2}$

[Centroid $\equiv$ orthocentre in equilateral]

$m_{PD} = \frac{-1}{m_{AB}} = \frac{-1}{-\frac{1}{2}} = 2$

$= \tan\theta \Rightarrow \frac{2}{\sqrt{5}}$, $\cos\theta = \frac{1}{\sqrt{5}}$

equation of pp′ is

$\frac{x - y}{\frac{1}{\sqrt{5}}} = \frac{y - \frac{3}{2}}{\frac{2}{\sqrt{5}}} = \pm \frac{\sqrt{5}}{2\sqrt{3}}$

$x = 4 \pm \frac{1}{2\sqrt{3}}$, $y = \frac{3}{2} \pm \frac{1}{\sqrt{3}}$

$G\left(4 + \frac{\sqrt{3}}{6}, \frac{3}{2} + \frac{\sqrt{3}}{3}\right)$, $G'\left(4 - \frac{\sqrt{3}}{6}, \frac{3}{2} - \frac{\sqrt{3}}{3}\right)$

$OG > OG' \Rightarrow \left(4 + \frac{\sqrt{3}}{6}, \frac{3}{2} + \frac{\sqrt{3}}{3}\right)$

Sol.52 C

P(2, 0), Q (4, 2)

line PQ is $x - y = 2$

$m_{PQ} = + 1$

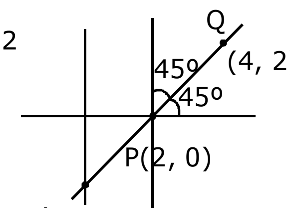
$\Rightarrow \theta = 45^o$

required line is

parallel to y-axis

(according questions)

$\Rightarrow$ $x = 2$



Sol.53 B

$x^2 - 4xy + 4y^2 + x - 2y - 6 = 0$

$(x - 2y + C)(x - 2y + d) = 0$

$(x - 2y)^2 + (C + d)x - 2(c + d)y + cd = 0$

$c + d = 1$, $cd = -6$

$c = 3$, $d = -2$

lines are $(x - 2y + 3) = 0$, $(x - 2y - 2) = 0$

distance = $\left|\frac{3 - (-2)}{\sqrt{1^2 + 2^2}}\right| = \frac{5}{\sqrt{5}} = \sqrt{5}$

Sol.54 A

$xy + 2x + 2y + 4 = 0$ & $x + y + 2 = 0$

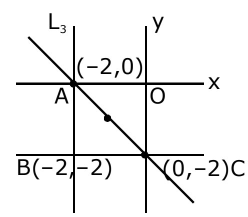
$(x + c)(y + d) = 0$

$xy + dx + cy + cd = 0$

$d = 2$, $c = 2$

$\frac{x + z = 0}{L_1}$ & $\frac{y + z = 0}{L_2}$

& $\frac{x + y + z = 0}{L_3}$



$L_1 \perp L_2$ L_2

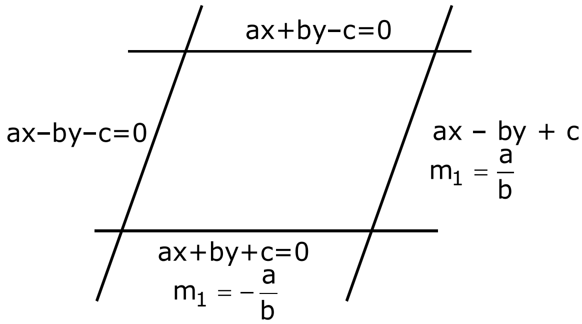
hypotaneous line L_3

mid point of hypotenous is circumcentre

$\left(\frac{0 - 2}{2}, \frac{-2 - 0}{2}\right) = (-1, -1)$

Sol.55 B

$ax \pm by \pm C = 0$



$m_1 = -\frac{a}{b}$, $m_2 = \frac{a}{b}$

$d_1 = -\frac{c}{b}$, $d_2 = \frac{c}{b}$

$d_1 = \frac{c}{b}$, $d_2 = -\frac{c}{b}$

Area of rhombus = $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{(m_1 - m_2)}\right|$

$= \left|\frac{2\frac{c}{b} \times \frac{2c}{b}}{2\frac{a}{b}}\right| = \frac{2c^2}{|ab|}$ sq. units

Sol.56 D

concurrent

$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$ a, b $\in$ R, a $\neq$ 1, b $\pm$ 1, c $\neq$ c

$C_2 \to C_2 \to C_1$ & $C_3 \to C_3 \to C_1$

$\Rightarrow$ a (b − 1) (c − 1) − (1 −a) (c − 1)
+1 (0 − (1 − a) (b − 1)) = 0

$\Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

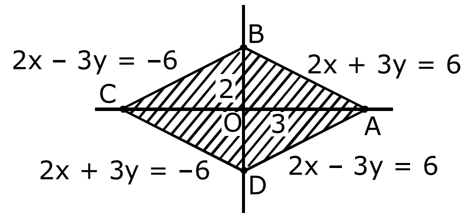
$\Rightarrow \left(1 + \frac{a}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 1$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

Sol.57 C

$2|x| + 3|y| \le 6$

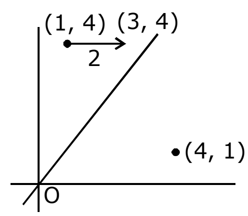
area ABCD = 4 (ΔOAB)

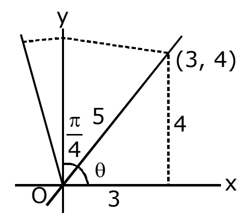


$= 4\left(\frac{1}{2}.2 \times 3\right) = 12$ sq. units

Sol.58 C

(i) Reflection about y = x of (4, 1) is (1, 4)





(ii) Now 2 units along (+) x direction
(1 + 2, 4 + 0) $\equiv$ (3, 4)

(iii) we wish to find

$$\left(5\cos\left(\theta + \frac{\pi}{4}\right), 5\sin\left(\theta + \frac{\pi}{4}\right)\right)$$

$x = 5\,\frac{\cos\theta}{\sqrt{2}} - \frac{5\sin\theta}{\sqrt{2}} = -\frac{1}{\sqrt{2}}$

$y = 5\,\frac{\sin\theta}{\sqrt{2}} + \frac{5\cos\theta}{\sqrt{2}} = \frac{7}{\sqrt{2}}$

$(x, y) \Rightarrow \left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$